MODELING MARKET- AND FIRM-LEVEL DEMAND FUNCTIONS IN COMPUTERIZED BUSINESS SIMULATIONS

STEVEN C. GOLD
THOMAS F. PRAY
Rochester Institute of Technology

During the past two years, ABSEL conferences have started to deal with design issues for simulation games. Both Goosen (1982) and Pray and Gold (1982) have addressed the need, by ABSEL members, to be more open about the design and the internal workings of simulations. Still, numerous questions by both new designers and users of computerized business games are often raised. Some typical questions include: how are the production processes modeled? how does one mathematically specify the market demand curve?

Users of existing games often want to modify the simulation to eliminate the conventional wisdom that often occurs after several semesters of play. Sometimes this may be easily accomplished through the use of control cards (or records) and variable parameters. Often, however, the modifications require alterations to the program itself. Thus, an appreciation for the internal workings of the simulation is needed. This understanding can help answer user-based questions such as: why was there such instability of price in this simulation? or why did all the firms spend so much on marketing and/or research and development?

AUTHORS' NOTE: An earlier version of this paper was presented at the 1982 ABSEL Conference in Tulsa, Oklahoma. This paper received the "Best Research" award at the conference and was printed in the 1983 proceedings.

SIMULATION & GAMES, Vol. 15 No. 3, September 1984 346-363

Business and management simulations, in particular, are modeled to represent the "real world" firm and market environments. Students are supposed to gain insight into the workings of the "real world" by participating in the simulation. As a result, it is necessary for the functions and algorithms contained within the simulation to be at least consistent with the economic, managerial, and financial relationships found in the business world. Although these underlying principles and theories explaining the "real world" phenomenon are well-known, the task of precisely modeling and qualifying these relationships in a simulation is not straightforward. Understanding the pros and cons of different functional forms can vastly facilitate the modeling process and ensure realistic simulation results. A proper appreciation by designers and users of the different modeling approaches can also prevent games from yielding unreasonable results (i.e., blowing up!).

This article presents and analyzes an effective method for modeling and simulating demand, often the key component of ongoing business games.

PURPOSE

One purpose of this article is to review the problems associated with market and firm level demand models currently used in eight business games as outlined by Brooks (1975); Darden and Lucas (1969); Edge et al. (1980); Gold et al. (1984); Henshaw and Jackson (1972); Pray and Strang (1981); Pray et al. (1984); and Smith et al. (1974).

Another purpose is to present a system of equations that embodies a number of key theoretical properties and practical issues including: (1) a multiplicative (power) industry demand function that incorporates the principle that the marginal impact of any variable, for example, advertising, on the total demand is not constant but is dependent on the level of other independent variables, such as prices; (2) variable elasticities for one or more of the independent variables; (3) permitting the introduction of diminishing returns on any of the independent variables; (4) eliminating the impact of irrational or faulty decision inputs on total market demand determination; (5) using exponential smoothing to recognize intertemporal movements in the decision variables; (6) having an intrinsic stockout adjustment routine which reallocates, in the same decision period, excessive stockouts to other firms in the industry; (7) employing a multiplicative firm level share equation that has variable price elasticity of demand.

Finally, our purpose is to demonstrate, with a numerical example, the power and flexibility of the demand system.

COMMON PITFALLS OF SIMULATING DEMAND

In a paper presented at ABSEL by Pray and Gold (1982), the authors investigated the underlying demand functions in eight contemporary simulations. Their analysis demonstrated that a number of the eight simulations investigated were unstable and that extreme decision values could induce unrealistic results. They also found that a number of the simulations inadequately incorporated diminishing returns, thus explaining why certain games induce nonprice competition such as continually increasing expenditures on market-related decisions. One game failed to adequately differentiate the firm-level from the industry-level demand which, in turn, could cause price instability. (The opposite of the well-known Sweezy Kinked Demand Theory for Oligopoly.) Pray and Gold (1982) concluded the following:

> It was found that many different modeling forms and techniques were employed by the simulation designers. Some of the simulations incorporated lagged stockouts in the demand function, while others introduced uncertainty, either in actual demand, or in stockout returns. Most of the simulations incorporated an intertemporal movement in demand analysis. The majority of these used exponential smoothing with larger values of the smoothing coefficient being applied to the price and marketing variables. As noted, certain of the demand functions were somewhat unstable and yielded unrealistic results, if left unconstrained. The designers,

in most cases, imposed constraints on the decision variables to prevent discrepancies between theory and simulation play. The following points summarize the key advantages and disadvantages to the three forms used:

> The Linear Demand Model permits variable elasticities. However, the impact of the marginal change in an independent variable is not related to the level of the other independent variables. Tentative elasticity analysis suggests the functional form is sensitive and the elasticities may vary rapidly. Input constraints should be imposed to insure realistic results.
>
> The Non Linear Model permits variable elasticities. Tentative analysis suggests it is difficult to separate out the impact of an individual decision on the demand. Highly unstable and contraints on the decision variables are needed.
>
> The Multiplicative Demand Function maintains a constant elasticity over the range of decision values. The impact of the marginal change in an independent variable is related to the level of the other independent variables. Appears to be stable at the industry level. However, at the firm level care must be taken to avoid "zero" level decisions.
>
> For stability of price (i.e., Sweezy Kink Demand Theory) care should be taken to insure that the firm level price elasticity be larger than the industry level. The "inverse kink" found in one simulation will probably induce instability in prices.
>
> The elasticities of marketing and Research and Development variables measure the degree of diminishing returns. They in turn suggest the relative importance of those variables in the decision process. The lack of adequate diminishing returns at the firm level, even with substantial diminishing returns at the industry level, induces non-price competition and may cause excessive expenditures on that decision variable.

This conclusion was the force behind this article and the development of a system of equations that may be employed to model both industry- and firm-level demand under many different conditions (and with a variety of decision variables). This system will remain stable and does not have the shortcomings found in other games.

A SUGGESTED SYSTEM FOR MODELING DEMAND

The system recommended for modeling demand is composed of four critical components: (1) conventional sample mean calculations for the independent and dependent variables, with the exception of the price variable (the harmonic mean should be employed to calculate the average market price); (2) exponential smoothing on all demand variables to capture the intertemporal effects; (3) a generalized multiplicative market demand function that allows for variable elasticities; and (4) a multiplicative firm-level demand function that has variable firm-level elasticities and is constrained by the total market demand. The suggested system is presented in Table 1, and consists of eight equations. Each of the principle components is described in detail below.

THE HARMONIC MEAN

The harmonic mean computes the average market price by weighting low prices relatively more than high prices. This property is desirable because low priced products (firms) generate higher quantities demanded than high priced firms. The formula used to calculate the harmonic mean is:

$$P = n / \sum_{i=1}^{n} (1/P_i) \quad [1]$$

where: n = the number of firms in the industry
P_i = the price of firm i
P = the average price for the industry

A simple example will illustrate the effect of using the harmonic mean. Suppose that the market consists of a duopoly situation with firm #1 charging $10 and firm #2 charging $20. The $15 conventional mean, implicitly assigning equal weights to each item, would overstate the "true" average price because the lower price actually induce more sales than the higher price. The harmonic mean calculation yields an average price of $13.33, which would more closely reflect the actual weighted average for the market price.

TABLE 1
A Suggested System for Modeling Demand

HARMONIC MEAN FOR AVERAGE PRICE

(1) $P = n / \sum_{i=1}^{n} (1/P_i)$

EXPONENTIAL SMOOTHING ON VARIABLES

(2) $P = aP_n + (1-a)P_o$; where $0<a<1$

(3) $M = bM_n + (1-b)M_o$; where $0<b<1$

(4) $R = cR_n + (1-c)R_o$; where $0<c<1$

An "o" subscript indicates a period-old smoothed value.
An "n" subscript indicates the most current mean value.

MARKET DEMAND

(5) $Q = g_1 P^{-(g_2+g_3P)} M^{+(g_4-g_5M)} R^{+(g_6-g_7R)}$

FIRM DEMAND

(6) $w_i = (P_i+k_i)^{-(k_2+k_3P_i)} (m_i+k_4)^{+(k_5-k_6m_i)} (r_i+k_7)^{+(k_8-k_9r_i)}$

(7) $s_i = w_i / \sum_{i=1}^{n} w_i$

(8) $s_{max} = f_i + 3\sqrt{f_i(1.-f_i)/n}$

(9) $q_i = s_iQ$

EXPONENTIAL SMOOTHING

The demand for a product depends not only on the current values of the independent demand variables, but also on their historical values. For instance, both current and past expenditures on advertising impact the sales potential of a firm. Exponential smoothing is a convenient technique allowing the simulation designers to specify the role and the importance of history on current demand. The conventional formulas are presented below with an example:

$$P = aP_n + (1\text{-}a)P_o; \text{ where } 0 < a < 1 \quad [2]$$
$$M = bM_n + (1\text{-}b)M_o; \text{ where } 0 < b < 1 \quad [3]$$
$$R = cR_n + (1\text{-}c)R_o; \text{ where } 0 < c < 1 \quad [4]$$

where: P = exponentially smoothed harmonic price
M = exponentially smoothed marketing expenditures
R = exponentially smoothed research and development expenditures

An "o" subscript indicates a period-old smoothed value, and an "n" subscript indicates the most current mean value.

The values of a, b, and c (the exponential smoothing coefficients) determine the impact of historical data on the current demand. Larger values for these coefficients put more weight on current data. For example, suppose that the value for b was .75. This means that a weight of .75 is assigned to the current average for marketing expenditures and the historical data is assigned the residual .25 weight in an exponentially declining fashion.

Larger values for the coefficients are equivalent to a smaller number of terms in a moving average. For instance if a is .5, this will have about the same effect as a moving average with four terms in it.[1] A value of .05 is roughly equivalent to a 39 period moving average. Both theory and evidence indicate that large values for a and b would be desirable, whereas a smaller smoothing value would be appropriate for variables such as research and development.

MARKET DEMAND

A three-variable market demand function which allows for both multiplicative demand properties and variable elasticities is specified below:[2]

$$Q = g_1 P^{(g_2+g_3P)} M^{+(g_4-g_5M)} R^{+(g_6-g_7R)} \quad [5]$$

where:

Q = the total market demand
P = the exponentially smoothed harmonic price
R = the exponentially smoothed research and development expenditures
M = the exponentially smoothed marketing expenditures
g_i = parameters or constants (for i = 1, 7)

The values assigned to the parameters ($g_1, g_2 \ldots g_7$) depend on the designer's specification concerning the elasticities (sensitivities) associated with the demand. The determination of the values of the seven parameters is discussed in detail in a forthcoming section of this article.

FIRM DEMAND

There are four basic components of the firm demand function: (1) the weighting function, (2) the market share equation, (3) the stockout routine, and (4) the quantity equation. Each of these are described below.

Weighting Function

The weighting function determines the magnitude of the value that is used to calculate the market share of the firm as a function of total market demand. The weighting function is a variable elasticity multiplicative function that is similar to the market demand equation previously specified. The weighting function suggested is as follows:

$$w_i = P_i+k_i^{-(k_2+k_3P_i)} \; m_i+k_4^{+(k_5-k_6m_i)} \; r_i+k_7^{+(k_8-k_9r_i)} \quad [6]$$

where:

w_i = weight of firm i
P_i = price of firm i
M_i = marketing expenditures for firm i

r_i = research and development expenditures for firm i
k_i = parameters or constants for i = 1, 9

The purpose of parameters k_1, k_4, and k_7 is to prevent the weight from equalling zero if a firm enters a zero decision for one of the demand variables. The magnitude of these parameters (k_1, k_4, and k_7) may be arbitrarily set at any small magnitude relative to the respective demand variables (P_i, m_i, or r_i).

Share Equation

The share equation is the weight of the firm (i.e., equation 6) divided by the sum of the weights for all firms in the market. The share equation if as follows:

$$s_i = w_i \Big/ \sum_{i=1}^{n} w_i \qquad [7]$$

where: s_i = market share for firm i
n = number of firms in the market
w_i = weight of firm i

Stockout Routine

If a firm behaves in an irrational fashion, causing them to receive an inordinate amount of the industry demand, and they are unable to supply the required goods, the stockouts (unsatisifed demand) are redistributed in the same period to the other firms in the industry via the forces of supply and demand.

Two criteria must be satisfied before a stockout condition is declared: (1) the firm's share, based on equation 7, is "too large" is defined by equation 8); (2) the firm cannot satisfy the demand. If both criteria are met, then the stockouts are redistributed to the other firms.

In reference to the first criteria, the maximum firm share in a given market is determined by using a quality control p-chart. The essence of the control chart is based on a firm's share being within three standard deviations of the expected share. Equation 8 demonstrates the upper control limit:

$$s_{max} = f_i + 3 \sqrt{f_i(1.-f_i) / n} \qquad [8]$$

where: f_i = 1/n and n is the number of firms in the market

s_{max} = upper limit on a reasonable share for a firm and n-firm market

In reference to the second criteria, although s_{max} indicates that the share is suspect, the routine then must ascertain whether or not the firm could supply the desired quantity. This may be accomplished by comparing the firm's demand potential, from equation 9, with their total finished goods available. If the share is beyond the s_{max} and the firm is unable to satisfy the demand, the routine reallocates the excessive demand to the other firms by normalizing equation 7, after removing the unrealistic firm's share.

Quantity Equation

The quantity demanded of firm i (q_i) is equal to the market share of firm i (s_i) multiplied by the total market demand (Q) from equation 5. The firm demand equation is as follows:

$$q_i = s_i Q \qquad [9]$$

The benefit of this approach is that it restricts the sum of the individual firm demands to equal the market demand determined by equation 5. As noted, if excessive stockouts occur to one or more firms, the stockout routine reallocates the unsatisfied demand to the other firms. This prevents the industry from being distorted by bad decisions or errors in data entry.

DERIVING THE ELASTICITIES

The elasticity associated with each demand variable is derived by applying the conventional formula:

$$E_{xi} = (dq/dx_i)\,(x_i/q)$$

where:
x_i = the demand variable i
E_{xi} = the elasticity of the demand variable i
q = the quantity demanded
dq/dx_i = the partial derivative of q with respect to x_i

The market- and firm-level elasticity equations for the demand function previously mentioned are given below. The firm elasticity is derived for the weighting function (equation 7).

PRICE ELASTICITIES

Market Level: $$E_P = g_2 + g_3P\,(1 + \ln P) \qquad [10]$$

Firm Level: $$E_{P_i} = k_2 + k_3[P_i + k_i]\,[1 + \ln(P_i + k_i)] \qquad [11]$$

Market and firm price elasticities in this demand system increase with increases in price, because it is assumed that all parameters are positive (i.e., all $g_i > 0$ and $k_i > 0$). Additionally, the rate of increase of the price elasticity with respect to increases in price level is also nonlinear. Furthermore, unlike the linear systems observed in 2, the price elasticity is independent of the other demand variables in the system that enhances the stability of the system.

MARKETING ELASTICITIES

Market Level: $$E_M = g_4 - g_5M(1 + \ln M) \qquad [12]$$

Firm Level: $$E_{m_i} = k_5 - k_6[m_i + k_4][1 + \ln(m_i + k_4)] \qquad [13]$$

Market and firm-level marketing elasticities decrease with increases in marketing expenditures because g_5 and $k_6 > 0$. As in the case of price elasticity, the market elasticity relationship is nonlinear and independent of other demand variables.

RESEARCH AND DEVELOPMENT ELASTICITIES

Market Level: $$E_R = g_6 - g_7R(1 + \ln R) \quad [14]$$

Firm Level: $$E_{r_i} = k_8 - k_9[r_i + k_7][1 + \ln (r_i + k_7)] \quad [15]$$

Because g_7 and k_9 are both assumed to be greater than zero, the research and development elasticities have the same properties as the marketing elasticities.

PARAMETER DETERMINATION

Solving the parameters in the demand system to obtain the desired elasticities simply involves the following three-step procedure:

(1) Select the starting value for each demand variable and the corresponding elasticity value.
(2) Select the second value (data point) for each demand variable and corresponding elasticity value.
(3) Substitute the elected values into the elasticity formulas and solve the simultaneous equations to calculate the required parameter values.

To assist in understanding how the system works a simple example is presented.

ILLUSTRATIVE EXAMPLE

An example of how to determine the parameter values for the market demand function is presented to illustrate the general

TABLE 2
Demand Variables and
Corresponding Elasticities

Price ($/unit)	Price Elasticity	Marketing ($)	Marketing Elasticity
$10.00	.05	$50,000	3.0
20.00	1.0	150,000	1.0

procedure and to demonstrate the properties of the function. It is assumed for simplicity that the desired demand function consists of only two independent variables, say price (P) and marketing expenditure (M). Furthermore, the designer has specified a priori the following values for the demand variables and the corresponding elasticities:

Substituting the values for price and price elasticity into the market-level price elasticity formula (equation 10), the following two equations are obtained:

$$0.5 = g_2 + 33.026 g_3; \quad \text{where } E_P = .5, \quad 33.026 = 10\,(1+\ln 10) \qquad [16]$$

$$1.0 = g_2 + 79.915 g_3; \quad \text{where } E_P = 1.0, \quad 79.915 = 20(1+\ln 20) \qquad [17]$$

Solving these two equations simultaneously, the values of g_2 and g_3 are:

$$g_2 = 0.15$$

$$g_3 = 0.01$$

Repeating this procedure and substituting the values for the marketing expenditures and marketing elasticities into the market-level elasticity formula (equation 12) yields the following two equations:

$$3.0 = g_4 - 590{,}990.0g_5; \quad \text{where } E_M = 3.0 \\ = 50{,}000\ (1 - \ln 50{,}000) \qquad [18]$$

$$1.0 = g_4 - 1{,}937.760.0g_5; \quad \text{where } E_M = 3.0 \\ = 150{,}000\ (1 + \ln 150{,}000) \qquad [19]$$

Solving equations 18 and 19 simultaneously, the values for g_4 and g_5 are:

$$g_4 = 3.88$$
$$g_5 = .0000015$$

Consequently, the illustrative market demand function in this example is:

$$Q = g_1 P^{-(0.01 + 0.15P)} M^{+(3.88 - 0.0000015M)}$$

The parameter g_1 is a scaling factor and may be arbitrarily assigned a value. It does not affect the elasticities. For this problem g_1 was assigned the value of 2.34×10^{-12}.

SIMULATING THE MARKET DEMAND FUNCTION

The market demand function derived in the above example will be simulated to illustrate the impact of price and marketing expenditures on the sales potential. More specifically, the simulation considers two cases: (1) the impact of variations in price on quantity demanded, holding marketing expenditures fixed at a starting value of $50,000; and (2) the impact of variations in marketing expenditures on quantity demanded, holding the price

TABLE 3
Impact of Price on Market Demand when Marketing Expenditures are Fixed at $50,000

PRICE	QUANTITY DEMANDED	PRICE ELASTICITY
10.00	997,228	0.50
12.00	906,603	0.57
14.00	824,933	0.66
16.00	750,791	0.75
18.00	703,249	0.85
20.00	621,492	1.00
22.00	564,065	1.05
24.00	513,468	1.15
26.00	466,291	1.26
28.00	423,172	1.36
30.00	283,788	1.47

variable fixed at the starting value of $10.00. The results of the simulation are reported in Tables 3 and 4.

As noted in Table 3, the price elasticity of demand increases slowly and steadily with increases in the average market price. The price elasticity is initially inelastic with a value of .5 (in absolute terms), at a starting price of $10.00. It increases to a unitary elastic value when price reaches $20.00, as specified a priori.

The marketing elasticity of demand, noted in Table 3, decreases relatively quickly with increases in the average marketing expenditure for the industry. As noted, marketing expenditures are highly elastic at the starting point of $50,000, and decreases to unity at an expenditure of $150,000. These values are consistent with those specified in the illustrative example.

It is interesting to note that the advertising elasticity in this example turns negative at an expenditure level of $210,000. This indicates that after some point ($210,000) increases in marketing will actually hurt market demand because of oversaturation.

This demand system is sufficiently flexible to permit or not permit turning points (i.e., inflection points) to occur in the

TABLE 4
Impact of Marketing Expenditures on Demand when Price is Fixed at $10.00

MARKETING EXPENDITURES	QUANTITY DEMANDED	MARKETING ELASTICITY
50,000	997,228	3.00
70,000	2,567,199	2.60
90,000	4,708,172	2.20
110,000	7,046,763	1.80
130,000	9,206,539	1.40
150,000	10,910,114	1.00
170,000	12,011,372	0.55
190,000	12,483,417	0.13
210,000	12,385,871	-0.30
230,000	11,828,334	-0.72
250,000	10,939,432	-1.16

function. These turning points can be readily specified by the simulation designer by following the three-step procedure outlined earlier.

SUMMARY AND CONCLUSION

This article represents an ongoing attempt to encourage open discussion concerning the design and development of computerized business simulations. Unfortunately, there appears to be a reluctance, even by ABSEL members, to discuss internal modeling components. In all the past ABSEL conferences, less than twenty professional papers dealt with design issues.

To encourage open discussion about modeling, this article developed a mathematical model of a demand system which may be used in designing a computerized business game. The demand system has been shown to possess a number of desirable properties:

(1) A harmonic mean to more effectively approximate average market price. A conventional sample mean has been shown to overstate the true average price.
(2) Exponential smoothing to capture intertemporal effects. The simulation designer, by specifying the smoothing coefficients, can easily control the role of history in the simulation.
(3) Multiplicative market- and firm-level demand functions that permit variable elasticities, diminishing (or increasing) returns, and stability. The parameters of the demand system have been shown to be easily solved once the desired elasticities are specified by the designer.
(4) A stockout routine which redistributes unsatisfied demand within the same period of simulation play. This prevents unrealistic market or industry results from occurring if a firm (or set of firms) makes economically irrational sets of decisions.

NOTES

1. When the average age of data is used as a criteria, the value of the smoothing coefficient may be equated to the number of terms in the moving average by: $N = (2-a)/a$, where N is the number of periods in the moving average and a is the exponential smoothing coefficient.
2. This model may be easily generalized to the n-variable case.

REFERENCES

BROOKS, L. D. (1975) Instructor's Manual—The Financial Management Decision Game. Richard D. Irwin Inc., Homewood, IL: Irwin.

DARDEN, W. R. and W. LUCAS (1969) The Decision Making Game—An Integrated Operations Management Simulation. New York: Prentice-Hall.

EDGE, A. G., B. KEYS, and W. E. REYMUS (1980) Instructor's Manual—The Multinational Game. Dallas: Business Publications Inc.

GOLD, S. C., T. F. PRAY, and T. DENNIS (1984) Instructor's Manual—MICROSIM—A Microeconomics Simulation. New York: MacMillan.

GOOSEN, K. R. (1982) A Generalized Algorithm for Designing and Developing Business Simulations. Proceedings of the Association of Business Simulations and Experiential Learning.

HENSHAW, R. C. and J. JACKSON (1972) The Executive Game. Homewood, IL: Irwin.

PRAY, T. and S. GOLD (1982) Inside the Black Box: An Analysis of Underlying Market and Demand Functions in Contemporary Business Simulations. Proceedings of the Association of Business Simulations and Experiential Learning.

PRAY, T. F., S. C. GOLD, D. BURLINGAME, and D. STRANG (1984) Instructor's Manual—DECIDE-P/OM: An Operations Management Simulation. New York: Random House.

PRAY, T. F. and D. STRANG (1981) Instructor's Manual—DECIDE. New York: Random House.

SMITH, W., N. E. ESTEY, and E. VINES (1974) Integrated Simulation. Cincinnati, OH: South Western Publishing.